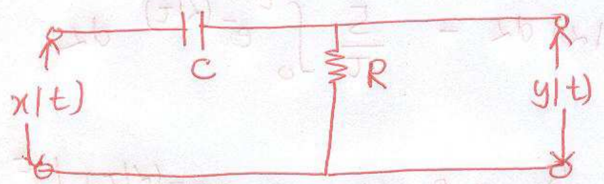
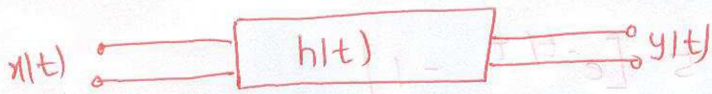
(1)

Solution of Assignment
Signals and Systems EC 402

The circuit diagram of the high pass filter is given as





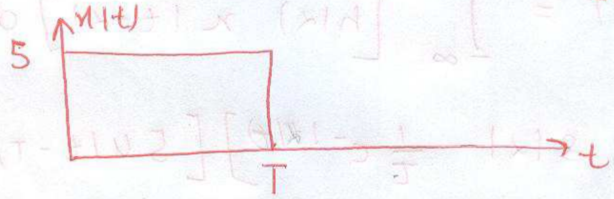
From the problem,

Impulse response of the circuit

$$h(t) = \left[\delta(t) - \frac{1}{\tau} e^{-(t/\tau)} u(t)\right]$$

Input

$$x(t) = 5\left[u(t) - u(t-T)\right]$$



First take the output response $y_1(t)$ of the network to the step signal $5u(t)$

$$y_1(t) = \int_{-\infty}^{+\infty} \left[x(\alpha)\ h(t-\alpha)\right] d\alpha$$

$$= \int_{-\infty}^{+\infty} \left[x(t-\alpha)\ h(\alpha)\right] d\alpha$$

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$$= \int_{-\infty}^{+\infty} \left[\delta(\alpha) - \frac{1}{\tau} e^{-(\alpha/\tau)} \right] \left[5u(t-\alpha) \right] d\alpha$$

$$= 5 \int_{0}^{t} \left[\delta(\alpha) - \frac{1}{\tau} e^{-(\alpha/\tau)} \right] d\alpha$$

$$= 5 \int_{0}^{t} \delta(\alpha)\, d\alpha - \frac{5}{\tau} \int_{0}^{t} e^{-(\alpha/\tau)}\, d\alpha$$

$$= 5 - \frac{5}{\tau} \times (-\tau) \times e^{-(\alpha/\tau)} \Big|_{0}^{t}$$

$$= 5 + 5 \left[e^{-t/\tau} - 1 \right]$$

$$\boxed{y_1(t) = \left[5e^{-(t/\tau)} \right]}$$

Now for the delay step signal $5u(t-T)$, the response is given as

$$y_2(t) = \int_{-\infty}^{+\infty} \left[h(\alpha)\, x(t-\alpha) \right] d\alpha$$

$$= \int_{-\infty}^{+\infty} \left[\delta(\alpha) - \frac{1}{\tau} e^{-(\alpha/\tau)} \right] \left[5u(t-T) \right] d\alpha$$

$$= 5 \int_{0}^{(t-T)} \left[\delta(\alpha) - \frac{1}{\tau} e^{-(\alpha/\tau)} \right] d\alpha$$

$$= 5 \int_{0}^{(t-T)} \delta(\alpha)\, d\alpha - \frac{5}{\tau} \int_{0}^{(t-T)} e^{-(\alpha/\tau)}\, d\alpha$$

$$= 5 - \frac{5}{\tau} \times (-\tau) \times e^{-\alpha/\tau} \Big|_{0}^{(t-T)}$$

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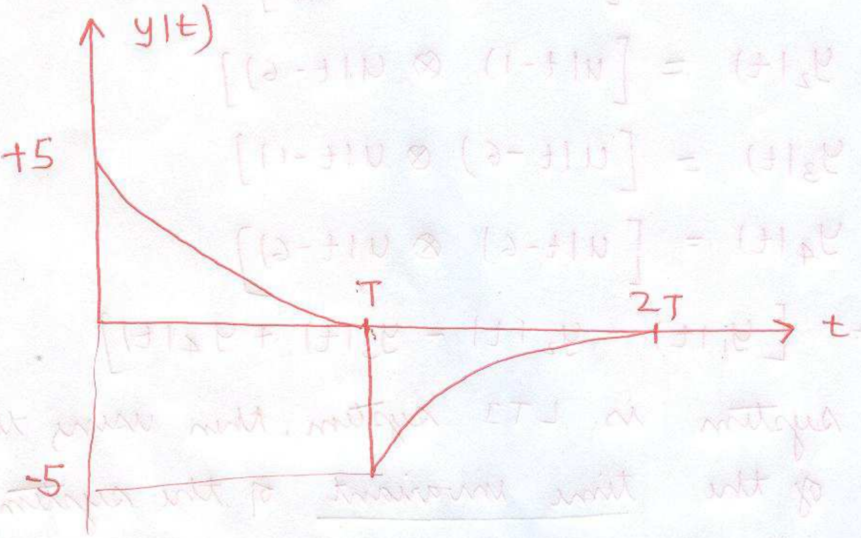
$= 5 + 5\left[e^{-(t-T)/\tau} - 1\right]$

$y_2(t) = 5e^{-(t-T)/\tau}$

Since system is LTI system. Then using the linearity property of the system

$y(t) = \left[y_1(t) - y_2(t)\right]$

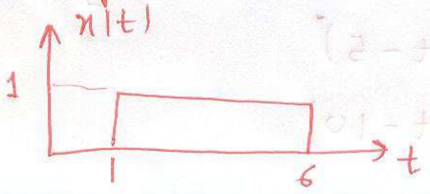
$\Rightarrow \quad y(t) = \left[5e^{-t/\tau}u(t) - 5e^{-(t-T)/\tau}u(t-T)\right]$

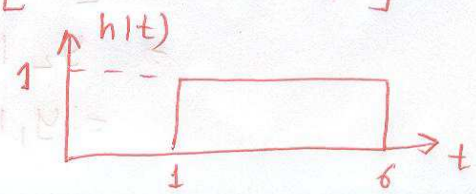


Q2) – From the problem

Input $x(t) = \left[u(t-1) - u(t-6)\right]$

Impulse response $h(t) = \left[u(t-1) - u(t-6)\right]$





The output response of the LTI system is given as (4)

$$y(t) = [x(t) \otimes h(t)]$$

$$= [u(t-1) - u(t-6)] \otimes [u(t-1) - u(t-6)]$$

$$= [\{u(t-1) \otimes u(t-1)\} + \{u(t-6) \otimes u(t-6)\} - \{u(t-6) \otimes u(t-1)\} - \{u(t-1) \otimes u(t-6)\}]$$

Let

$$y_1(t) = [u(t-1) \otimes u(t-1)]$$

$$y_2(t) = [u(t-1) \otimes u(t-6)]$$

$$y_3(t) = [u(t-6) \otimes u(t-1)]$$

$$y_4(t) = [u(t-6) \otimes u(t-6)]$$

$$y(t) = [y_1(t) - y_2(t) - y_3(t) + y_4(t)]$$

Since the system is LTI system, then using the property of the <u>time invariant</u> of the system we can write the $y_2(t)$, $y_3(t)$ and $y_4(t)$ in the terms of $y_1(t)$ as

$$y_2(t) = [u(t-1) \otimes u(t-6)]$$

$$y_2(t) = y_1(t-5)$$

$$y_4(t) = [u(t-6) \otimes u(t-6)]$$

$$= y_2(t-5)$$

$$= y_1(t-10)$$

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$$y_2(t) = [u(t-1) \otimes u(t-6)]$$
$$= y_1(t-5)$$

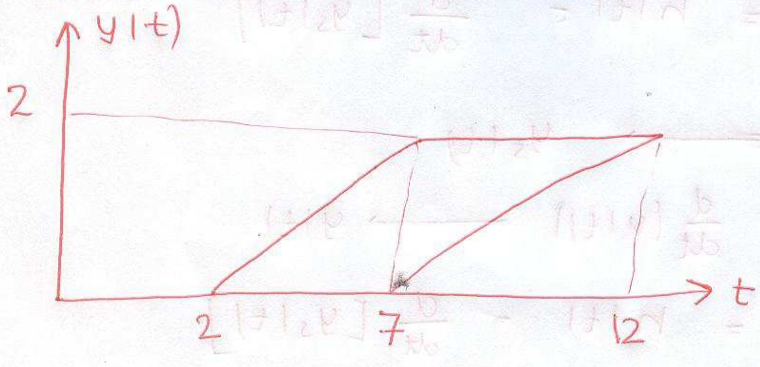
Now calculating

$$y_1(t) = [u(t-1) \otimes u(t-1)]$$
$$= \int_{-\infty}^{+\infty} [u(\alpha-1)\, u(t-\alpha-1)]\, d\alpha$$
$$= \int_{1}^{(t-1)} d\alpha$$

$$\boxed{y_1(t) = (t-2)\, u(t-2)}$$

$$y(t) = [(t-2)u(t-2) - 2(t-7)u(t-7) + (t-12)u(t-12)]$$

$$\boxed{y(t) = [r(t-2) - 2r(t-7) + r(t-12)]}$$



3) Resultant output is given as

$$y_s(t) = \left[\frac{1}{2} t\,u(t) - \frac{1}{20}(1-e^{-10t})\,u(t)\right]$$

Since the unit step signal is applied to the input of the signal and $y(t)$ is recorded as the output. When the impulse signal is applied to the input of the system. then the output of system is the impulse response of the system $[h(t)]$.

$$u(t) \longrightarrow \boxed{h(t)} \longrightarrow y_s(t)$$

$$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = h(t)$$

$$y(t) = h(t) = \frac{d}{dt}[y_s(t)]$$

$$u(t) \longrightarrow y_s(t)$$

$$\delta(t) = \frac{d}{dt}(u(t)) \longrightarrow y(t)$$

So $$y(t) = h(t) = \frac{d}{dt}[y_s(t)]$$

$$= \frac{d}{dt}\left[\frac{t\,u(t)}{2} - \frac{1}{20}(1-e^{-10t})\,u(t)\right]$$

$$y(t) = \frac{1}{2}\left[1 - e^{-10t}\right] \quad t \geq 0$$ 7

Impulse response of the system

$$\boxed{h(t) = \frac{1}{2}\left[1 - e^{-10t}\right]}$$

5) Impulse response of the LTI system

$$h(t) = \left[3\delta(t-1)\right]$$

$$H(s) = L[h(t)]$$

$$= L\left[3\delta(t-1)\right] = \int_{-\infty}^{+\infty} 3\delta(t-1)e^{-st}\,dt$$

$$\boxed{H(s) = 3e^{-s}}$$

where $s = (\sigma + j\omega)$

Now input $x(t) = 5e^{j70t}$

We can write this input signal for the LTI system as

$$x(t) = Xe^{st}\Big|_{X=5\;;\;s=j70}$$

$$= Xe^{j\omega t}\Big|_{X=5\;;\;\omega=70}$$

$$H(s) = 3e^{-s}$$

Steady state response can written as

$$y(t) = Xe^{st}\,H(s)\Big|_{s=j70\,:\,X=5}$$

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$$= 5e^{j70t}\,3e^{-j70}$$

$$\boxed{y(t) = 5e^{j70(t-1)}}$$

Similarly $x(t) = 7e^{-j120}$

$$H(s) = 0$$

$$y(t) = Xe^{st}\,H(s)\Big|_{X=7;\ W=-j120}$$

$$y(t) = 0$$

From the RC circuit the response is given as

$$h(t) = \left[\delta(t) - \frac{1}{\tau}e^{-(t/\tau)}u(t)\right]$$

$$F[h(t)] = \int_{-\infty}^{+\infty}\left[\delta(t) - \frac{1}{\tau}e^{-(t/\tau)}u(t)\right]e^{-jwt}dt$$

$$= \int_{-\infty}^{+\infty}\left[\delta(t)e^{-jwt}\right]dt - \frac{1}{\tau}\int_{-\infty}^{+\infty}e^{-(t/\tau)}e^{-jwt}u(t)dt$$

$$= 1 - \frac{1}{\tau}\int_0^{\infty}e^{-(\frac{1}{\tau}+jw)t}\,dt$$

$$= 1 + \frac{1}{\tau}\frac{1}{(\frac{1}{\tau}+jw)}\,e^{-(\frac{1}{\tau}+jw)t}\Big|_0^{\infty}$$

$$= 1 + \frac{1}{1+jw\tau}\left[0-1\right]$$

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$$= \left[\frac{j\omega\tau}{1+j\omega\tau}\right]$$

$\tau = 0.1$

$$H(j\omega) = \left[\frac{j\omega}{10+j\omega}\right]$$

(b) Input signal

(I) $x(t) = 5 \Big|_{X=5;\ s=0}$

Steady state response for the input signal is written as

$$y(t) = X e^{st} H(s) \Big|_{X=5;\ s=0}$$

$$= X e^{(\sigma+j\omega)t} H(j\omega) \Big|_{X=5;\ \omega=0}$$

$$= \left[5 \times e^{0t} \times \left(\frac{0}{10+0}\right)\right]$$

$$y(t) = 0$$

(II) $x(t) = \left[5 e^{j100t}\right]$

$$= X e^{st} \Big|_{X=5;\ \omega=100}$$

$$H(j\omega) = \frac{100j}{10+100j} = \left[\frac{10j}{1+10j}\right]$$

$$y(t) = X e^{(\sigma+j\omega)t} H(j\omega) \Big|_{X=5;\ \omega=100}$$

$$= \left[5 e^{j100t} \frac{10j}{1+10j}\right]$$

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$$y(t) = \left[4.975\, e^{j\,5.7}\, e^{j100t}\right]$$

$$y(t) = X\, H(jw)\, e^{(\sigma + jw)t}\Big|_{X=5;\ w=-100}$$

$$= \left[5\, e^{-j100t}\left[\frac{-j100}{10-j100}\right]\right]$$

$$y(t) = 4.975\, e^{-j5.7}\, e^{j100t}$$

$$x(t) = 10\cos 100t$$

$$= 5\left[e^{j100t} + e^{-j100t}\right]$$

$$y(t) = H(jw)\, X\, e^{(\sigma + jw)t}\Big|_{X=5;\ w=100;\ -100}$$

$$y(t) = 9.95\cos(100t + 5.7)$$

$$h(n) = a^n u(n)$$

$$x(n) = u(n)$$

Steady state response is given as

$$y(n) = \sum_{K=-\infty}^{+\infty}\left[x(K)\, h(n-K)\right]$$

$$= \sum_{K=-\infty}^{+\infty}\left[u(K)\, a^{(n-K)}\, u(n-K)\right]$$

$$= \sum_{K=0}^{+\infty}\left[a^{(n-K)}\right]$$

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$$= a^n \sum_{k=0}^{0} a^{-k}$$

$$a^n [1 + a^{-1} + - - - + a^{-n}]$$

$$y(n) = a^n \left[\frac{1 - a^{-(n-1)}}{1 - a^{-1}} \right]$$

Since the response depends only on the present and past values of the input with $x[n] = \delta[n]$, we obtained the response samples for each value of n in $h[n]$

$n=0$ $\quad h[0] = \frac{1}{3} [\delta[n] + \delta[n-1] + \delta[n-2]] \Big|_{n=0}$

$$h[0] = \frac{1}{3}$$

$$h[1] = \frac{1}{3} [\delta(1) + \delta(0) + \delta(-1)]$$

$$= \frac{1}{3}$$

$$h[2] = \frac{1}{3} [\delta(2) + \delta(1) + \delta(0)]$$

$$= \frac{1}{3}$$

For $\quad n \geq 3$

$$h[n] = 0 \Big|_{n \geq 3}$$

$$h[n] = \frac{1}{3} [\delta(n) + \delta(n-1) + \delta(n+2)]$$

(12)

System is given as

$$y[n] = \left[x[n] + \frac{1}{2}\{x[n-1] + x[n+1]\}\right]$$

Applying

$$x[n] = \delta[n]$$

$$y[n] = \left[\delta[n] + \frac{1}{2}\{\delta(n-1) + \delta(n+1)\}\right]$$

It is clear that $h[n] \neq 0$ for $n < 0$

So system is noncausal